$\Delta-$STRING: A HYBRID BETWEEN EINSTEIN'S AND STRING PARADIGMES

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A. Einstein had an idea that the electron should have an inner structure which allows us to avoid singularities of point-like particles in classical and quantum electrodynamics. Einstein's-Wheeler's idea about this structure is that the electron is a wormhole: a bridge connecting two remote parts of a Universe. The picture for this point of view is presented on Fig.1a.

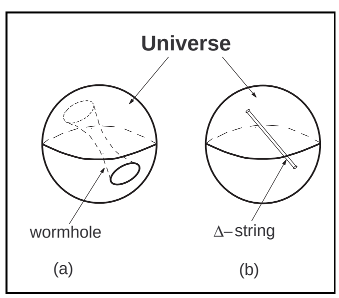

Figure 1. The wormhole **(a)** or thin gravitational flux tube **(b)** attached to remote parts of a single Universe.

One of the bad peculiarities of this point of view is that the wormhole has a curvature in the same order as an entire part of the Universe. It means that the linear sizes of the wormhole can be compared with the sizes of the Universe and the whole object = Universe + wormhole is too curved. But may be, the picture presented on Fig.1b is more attractive. Here the cross section of the wormhole is

*Work supported by Alexander von Humboldt Foundation.
†Valuable comments by E. Mielke are gratefully acknowledged. Thanks the Deutsche Forschungsgemeinschaft DFG for financial support.

of the order of the Planck length. In this case, the external part of the Universe is as it is, and the attachment points are like to (+) and (−) electric charges. We see that such a bridge is more similar to a string attached to a D-brane.

Thus, our aim is to show that in vacuum 5D gravity there exist wormhole-like solutions with a superthin and superlong mouth. We call such a wormhole a Δ-string with the following motivation for the words Δ and string: (i) "string" means that it is very thin and very long, (ii) Δ means that the attachment point is like a delta of a river in the consequence of a spacetime foam. In other words, the attachment point of the Δ-string is spread.

The initial equations for the description of the Δ-string are ordinary 5D Einstein's equations. The detailed investigation of all spherically symmetric solutions tells us that the solutions depend on the relation between electric q and magnetic Q charges, and the case most interesting for us is $(1 - Q/q) << 1$ where $Q < q$. The numerical investigation shows us that in this case the solution is similar to Fig.1b: there is an arbitrarily long mouth with the length $l \to \infty$ by $\delta \to 0$.

The numerical and approximate calculations [1 2] show that: (i) $a(r_H) = 2a(0)$, r_H is defined from the relation: $ds^2(\pm r_H) = 0$; (ii) $l(\delta) \approx a(0) \ln \delta$. The cross section of the Δ-string of the origin $a(0)$ is arbitrary, and can be chosen $a(0) \approx l_{Pl}$. Thus the mouth of the 5D spherically symmetric wormhole-like solution with $q \approx Q$ $(q > Q)$ can be superthin and superlong, and consequently can be considered as a string-like object, namely a Δ-string.

It is necessary to note that one can insert this part of a flux tube solution between two Reissner-Nordström black holes. This situation is much more interesting because it is like to a string attached to 2 D-branes : the flux tube solution is the string, each Reissner-Nordström solution is D-brane and joining on the event horizon takes place.

Next, we would like to reduce our initial 5D Lagrangian to a 2D Lagrangian. At this step we set the sizes of 5^{th} and S^2 dimensions $\approx l_{Pl}$. The first step is the usual 5D $\to$ 4D Kaluza-Klein dimensional reduction. The further step is the reduction from 4D to 2D. We consider the region of spacetime where the topology is $M^2 \times S^2 \times S^1$ and the linear sizes of S^2 $(\theta, \varphi coordinates)$ are $\approx l_{Pl}$. We can give arguments that in this case all physical fields do not depend on the coordinates on the sphere S^2. The 4D metric can be expressed as

$$d \overset{(4)}{s}{}^2 = g_{\mu\nu} dx^\mu dx^\nu = g_{ab}(x^c) dx^a dx^b + \chi(x^c) \left(\omega^{\bar{i}} + B^{\bar{i}}_a(x^c) dx^a\right) \left(\omega_{\bar{i}} + B_{\bar{i}a}(x^c) dx^a\right) \tag{1}$$

where $a, b = 0, 1$; x^a are the time and longitudinal coordinates; $-\omega^{\bar{i}} \omega_{\bar{i}} = dl^2$ is the metric on the 2D sphere S^2. The dimensional reduction to 2 dimensions is

$$\overset{(4)}{R} = \overset{(2)}{R} + R(S^2) - \frac{1}{4} \Phi^{\bar{i}}_{ab} \Phi^{ab}_{\bar{i}} - \frac{1}{2} h^{ij} h^{kl} \left(D_a h_{ik} D^a h_{jl} + D_a h_{ij} D^a h^{kl}\right) - \nabla^a \left(h^{ij} D_a h_{ij}\right) \tag{2}$$

where $\overset{(2)}{R}$ is the Ricci scalar of 2D spacetime; D_μ and $\Phi^{\bar{i}}_{ab}$ are, respectively, the covariant derivative and the curvature of the principal connection $B^{\bar{i}}_a$, and $R(S^2)$ is the Ricci scalar of the sphere S^2; h_{ij} is some metric.

Let us consider the situation with the electromagnetic fields A_μ and $F_{\mu\nu}$.

$$A_\mu = \{A_a, A_i\} \ A_a \text{ is the vector; } A_i \text{ are 2 scalars;} \tag{3}$$

$$F_{ab} = \partial_a A_b - \partial_b A_a \text{ is the Maxwell tensor for } A_a; \tag{4}$$

$$F_{ai} = \partial_a A_i, \tag{5}$$

$$F_{ij} = 0 \tag{6}$$

here we took into account that $\partial_i = 0$.

Connecting all results we see that only the following *physical fields* on the (Δ-string) are possible : 2D metric g_{ab}, gauge fields $B^{\bar{i}}_a$, vectors A_a, tensors $F_{ab}, F_{a\bar{i}}, \Phi^{\bar{i}}_{ab}$, scalars χ and ϕ.

The next remark is that Δ-string can have an essential decreasing of the initial degrees of freedom. For example, $G_{\theta\theta}$ and $G_{\phi\phi}$ describe the lengths in the transversal directions but these directions are in the Planck region and consequently these degrees of freedom should be excluded from the effective Lagrangian: $\sqrt{-G}R_{(5)} \to L_{eff}$ with the loss of some degrees of freedom.

In conclusion, the Δ-string has the next very interesting properties

(1) In during of one of the paradigm of quantum gravity that the spacetime has not any structure by the length less Planck length we see that some vacuum solutions in the Kaluza-Klein gravity actually do not have any transversal structure, i.e. the Δ-string is really a 1-dimensional object.
(2) The Δ-string has a flux of electric field and consequently for an external observer living on the D-brane it looks like to a spread of electric charge.
(3) The Δ-string has the mixed classical and quantum properties: in the longitudinal direction it has classical properties but in transversal a quantum one.
(4) The Δ-string is an object where we can investigate the 2D spacetime foam.

The next investigations in this direction will be based on the 7D metrics presented in [3]-[4].

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